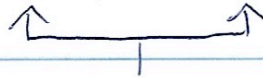


The process in lab frame is.

$$(\gamma m_{\pi} c, p) + (m_n c, 0) \rightarrow (\gamma_K m_K c, p_K) + (\gamma_{\Lambda} m_{\Lambda} c, p_{\Lambda})$$

Subject to  $p = p_K + p_{\Lambda}$ .

In the COM frame, it's.

$$(\gamma_1 m_{\pi} c, \gamma_1 m_{\pi} v_1) + (\gamma_2 m_n c, \gamma_2 m_n v_2) \rightarrow (m_K c, 0) + (m_{\Lambda} c, 0)$$


we made the 2 resultant particles at rest to minimize the total energy of the resultant system.

The problem asks to solve for  $(\gamma m_{\pi} c + m_n c)$  in lab frame, Lorentz invariance demands

$$\underbrace{(\gamma m_{\pi} c + m_n c)^2 - p^2}_{\text{lab frame}} = \underbrace{(m_K c + m_{\Lambda} c)^2}_{\text{COM frame}}$$

This equation has only 1 undetermined variable since  $p$  and  $\gamma$  are related algebraically, so it's solvable. Thus we have reduced the problem to an algebraic eq. to be solved.

rewriting  $p = \gamma m_{\pi} v$  gives.

$$(\gamma m_{\pi} c + m_n c)^2 - (\gamma m_{\pi} v)^2 = (m_K c + m_{\Lambda} c)^2$$

$$(\gamma m_{\pi} + m_n)^2 - (\gamma m_{\pi} \beta)^2 = (m_K + m_{\Lambda})^2$$

$$\beta^2 \gamma^2 = \gamma^2 \left[ 1 - \frac{1}{\gamma^2} \right] = \gamma^2 - 1$$

$$\Rightarrow (\gamma m_{\pi} + m_n)^2 - m_{\pi}^2 (\gamma^2 - 1) = (m_K + m_{\Lambda})^2$$

$$(\gamma m_{\pi} + m_n)^2 - \gamma^2 m_{\pi}^2 + m_{\pi}^2 = (m_K + m_{\Lambda})^2$$

Let  $x = \gamma m_{\pi} + m_n$

$$x^2 - (x - m_n)^2 = (m_K + m_{\Lambda})^2 - m_{\pi}^2$$

$$2x m_n = (m_K + m_{\Lambda})^2 - m_{\pi}^2 + m_n^2$$

$$x = \frac{1}{2m_n} \left[ (m_K + m_{\Lambda})^2 - m_{\pi}^2 + m_n^2 \right]$$

$$E_{\text{threshold}} = \frac{1}{2m_n} \left[ (m_K + m_{\Lambda})^2 - m_{\pi}^2 + m_n^2 \right] c^2$$

using  $m_{\pi} = 139.6 \text{ MeV}/c^2$ ,  $m_K = 494 \text{ MeV}/c^2$ ,  $m_n = 939.6 \text{ MeV}/c^2$ ,  $m_{\Lambda} = 1116 \text{ MeV}/c^2$ ,

$$E_{\text{threshold}} \approx \boxed{1838.19 \text{ MeV}}$$