Goldstein 7.20

The process in lab frame is.

$$(\gamma m_\pi c, p) + (m_n c, 0) \to (\gamma_K m_K c, p_K) + (\gamma_\Lambda m_\Lambda c, p_\Lambda)$$

subject to $\quad p = p_K + p_\Lambda .$

In the COM frame, it's.

$$(\gamma_1 m_\pi c, \gamma_1 m_\pi v_1) + (\gamma_2 m_n c, \gamma_2 m_n v_2) \to \underbrace{(m_K c, 0) + (m_\Lambda c, 0)}.$$

we made the 2 resultant particles at rest to minimize the total energy of the resultant system.

The problem asks to solve for $(\gamma m_\pi c + m_n c)$ in lab frame, Lorentz invariance demands

$$\underbrace{(\gamma m_\pi c + m_n c)^2 - p^2}_{\text{lab frame}} = \underset{\substack{\uparrow \\ \text{COM frame.}}}{(m_K c + m_\Lambda c)^2}$$

This equation has only 1 undetermined variable since p and γ are related algebraically, so it's solvable. Thus we have reduced the problem to an algebraic eq. to be solved.

rewriting $p = \gamma m_\pi v$ gives.

$$(\gamma m_\pi c + m_n c)^2 - (\gamma m_\pi v)^2 = (m_K c + m_\Lambda c)^2$$

$$(\gamma m_\pi + m_n)^2 - (\gamma m_\pi \beta)^2 = (m_K + m_\Lambda)^2$$

$$\gamma^2\beta^2 = \gamma^2\left[1 - \frac{1}{\gamma^2}\right] = \gamma^2 - 1$$

$$\Rightarrow \quad (\gamma m_\pi + m_n)^2 - m_\pi^2(\gamma^2 - 1) = (m_K + m_\Lambda)^2$$

$$(\gamma m_\pi + m_n)^2 - \gamma^2 m_\pi^2 + m_\pi^2 = (m_K + m_\Lambda)^2$$

Let $x = \gamma m_\pi + m_n$

$$x^2 - (x - m_n)^2 = (m_K + m_\Lambda)^2 - m_\pi^2$$

$$2x m_n = (m_K + m_\Lambda)^2 - m_\pi^2 + m_n^2$$

$$x = \frac{1}{2m_n}\left[(m_K + m_\Lambda)^2 - m_\pi^2 + m_n^2\right]$$

$$E_{threshold} = \frac{1}{2m_n}\left[(m_K + m_\Lambda)^2 - m_\pi^2 + m_n^2\right]c^2$$

using $m_\pi = 139.6\ MeV/c^2$, $m_K = 494\ \frac{MeV}{c^2}$, $m_n = 939.6\ \frac{MeV}{c^2}$, $m_\Lambda = 1116\ \frac{MeV}{c^2}$,

$$E_{threshold} \simeq \boxed{1838.19\ MeV}$$

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